MUSIC-TIME and CLOCK-TIME SIMILARITIES UNDER TEMPO CHANGE

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1. INTRODUCTION

At the last meeting of this group, we presented a general method and a set of subroutines for relating "music-time" or "PASS1" and "clock-time" or "PASS2" scores (**1). This paper will continue that investigation, emphasizing certain characteristics of the music-time score which are left unchanged under the operations of linear and equal-ratios tempo modification. We will use the terms and models developed in our earlier paper. Thus we will speak of:

1. beats as a property of music-time scores (i. e. a half note gets two beats);

2. duration in seconds as a property of clock-time scores (i. e. a half note lasts .8 seconds);

3. the clock factor of a beat point as the period of the tempo at that beat point. Thus the clock factor is the time, in seconds, that one beat would have at the given tempo. The clock factor associated with a tempo of 60 is 1; with 120, .5; with 600, .1; and

4. the duration of some passage in seconds as the integral of the clock factor curve of the passage with respect to beats.

A complete set of formulas for computing tempo and duration using several common curves is given below. Note that the second formula of each group is for duration and involves the integration of the clock-factor curve associated with the first formula of each group.

Sections 1-8 and the Appendices of this article were written by John Rogers in consultation with Philip Batstone, whose musical ideas provided one of the key motivations for this effort, and John Rochstroh, whose mathematical expertise provided the essential framework for our work. We are also indebted to Robert Carrier of the Research Computer Center of the University of New Hampshire for general help and advice in the preparation of these sections, particularly Appendices I and II. Section 9 was written by Philip Batstone, but makes use of concepts and programming techniques developed by the other two authors. We refer readers who are interested in further study of Prof. Batstone's compositional techniques to his doctoral dissertation (**2,**3).

```
1.  LINEAR TEMPO (hyperbolic clock factor)
    1.  T(bp)=(K * bp) + T1
    2.  DUR(bp)=(60/K) * LN(T(bp)/T1)

2.  EQUAL-RATIOS TEMPO and CLOCK FACTOR
    1.  T(bp)=T1 * ((T2/T1)**(bp/B))
    2.  DUR(bp)=(60/T1)*(B/LN(T1/T2))*((T1/T2)**(bp/B)-1.)

3.  INVERSE EQUAL-RATIOS TEMPO
    1.  T(bp)=T1 + T2 - T2*(T1/T2)**(bp/B)
    2.  DUR(bp)=((60*B)/((T1+T2)*LN(T1/T2)))*((LN(T1/T2)**(bp/B))-
        LN(T1+T2-T2*(T1/T2)**(bp/B)/T1))

4.  HYPERBOLIC TEMPO (linear clock factor)
    1.  T(bp)=(B*T1*T2)/(B*T2 + bp*(T1-T2)
    2.  Dur(bp)=60 * (bp/T1 + (bp**2/2*B)*((1/T2)-1/T1))
```

where
1. bp is some beat point,
2. T(bp) is the tempo at some beat point,
3. DUR(bp) is the duration in seconds at some beat point,
4. T1 is the initial tempo,
5. T2 is the final tempo,
6. B is the total number of beats in the passage,
7. K=(T2-T1)/B,
8. LN is the log, base e.

2. FIVE CRITERIA for RHYTHMIC SIMILARITY

When the tempo of a passage is absolutely steady, the ratio of the clock times of successive beats is 1:1 and the ratios of the actual durations of the notes in the "clock-time" score are the same as the ratios of the durations of the notes in the music-time score. If a continuous, monotonic curve of tempo change is applied, the clock times of successive beats change continuously in length and their ratio is, obviously, no longer 1:1. Clearly, the durations and the ratios of the durations of the notes in the clock-time score will no longer match those in the music-time score. There are five criteria we have found to be helpful when evaluating the similarities of passages undergoing tempo change to passages played at a steady tempo.

1. The Shape of Tempo Change vs the "Straight Line"

Consider the tempi and clock times of successive beats in relation to the starting tempo (T1) and its associated clock factor (CF1) and the ending tempo (T2) and its associated clock factor (CF2). A curve which diverges slowly from T1 produces a realization which diverges slowly from a steady, T1 realization of the notated passage. A curve which moves rapidly towards the goal tempo produces a realization which moves rapidly towards a steady, T2 realization of the notated passage. Perhaps the simplest criteria for evaluating tempo change is to compare the curve the tempo change function produces to the straight lines which would be produced by steady tempo realizations at either of the two specified tempi.

2. The "Equal-To","Greater-Than", and "Less-Than" Relationships

A second criteria has to do with the relationships equal-to (EQ.), greater-than (GT.), and less than (LT.). All tempo change functions destroy the EQ. relationship: notes of equal written value are not of equal clock-time value. Their clock-time duration depends on the tempo-change function being used and their place in the function. Some tempo change functions preserve the GT.-LT. relationship and some do not. Consider a case of a music-time rhythm of a dotted eighth followed by a sixteenth. The dotted eighth has a greater beat value than does the sixteenth. Suppose the tempo is slowing. If the ritard is great enough -- if the relatichnsip between the beginning tempo and the end tempo is large enough -- then the clock time durations produced will not have the same kind of relationship as the beat durations. For example, we could choose a ritard that would make the clock-time duration associated with the sixteenth note EQ. or GT. the clock time associated with the dotted eighth. These anomalous situations occur when (1) the "direction" of notated rhythm contradicts the "direction" of tempo change -- in our case the direction of notated rhythm was long to short while the direction of tempo change was fast (or short) to slow (or long) and (2) the difference in the given tempi is large in relationship to the change in notated values. If our music-time rhythm had been a whole note followed by a sixteenth note, it would have taken a much more extreme tempo curve to cause the sixteenth note to take the longer time.

Tempo change functions which preserve the GT.-LT. relationship are intuitively simpler and of more general use. If a significant number of the durational relationships of the music-time score were found to be reversed in the clock-time score, serious questions would be raised about the appropriateness of the tempo function being applied and the relationship of the performed piece to the notated one. It is also relevant for our purposes to note here that neither the equal-ratios nor the linear function preserves the GT.-LT. relationship for all cases.

3. The Maintainance of Rhythmic Proportionality

A third criteria concerns the proportionality of rhythms. Since, in a steady tempo, the ratios of successive beat durations is 1:1, all steady tempo performances of a passage "sound the same", in a sense. In its most obvious sense, proportionality of notated values is lost under tempo change. The relationship of a dotted eight to a sixteenth is different depending on where they are located. The ratio of a dotted eight followed immediately by a sixteenth is different from a dotted eight to a sixteenth two beats later. There is, however, a sense in which a certain "motivic" aspect of proportionality may be maintained. The proportionality of a dotted eightt followed by sixteenth would no longer be 3:1 and the GT. relationship would not necessarily be preserved, but we might expect that wherever this "motive" is found on the curve, it would be the same proportion. Further, the relationship of a dotted eighth to a sixteenth two beats later would not be 3:1, would not be the same as a dotted eighth followed immediately by a sixteenth, and would not necessarily preserve the GT relationship; but it is possible to make it the same as the relationships between all other dotted eighths to sixteenths following two beats later. The equal-ratios curve is the only tempo function that preserves this important aspect of proportionality, and that is one reason most tempo change programs have used it as the "base function". Other functions produce differing proportions depending on where the passage is located on the curve. Thus the rhythmic motive of a dotted eighth followed by a sixteenth would produce differing clock ratios depending on its locations in a musical passage with non-equal-ratios curves. In this sense, non-equal-ratios curves produce passages which "sound different" depending on their tempo.

There is a second aspect of rhythmic proportionality which must be mentioned. Suppose we have, in a steady tempo, two voices both stating the same rhythmic proportions, one twice as fast as the other. There is no continuous function that guarantees rhythmic proportionality of these voices under tempo change. Some ramifications of this problem will be discussed later in connection with mensuration canons.

4. Constant Duration and Tempo Change

A fourth kind of judgement about rhythmic similarity has to do with passages whose numbers of beats and tempi are related by some constant. Suppose we have a 10-beat passage at tempo 60 followed by a 20 beat passage at tempo 120. The number of beats is doubled but so is the tempo. Each passage will take the same amount of clock time. In general, if the number of beats in the passage and its tempo are both multiplied by the same constant, the clock time duration of each passage is the same. We now add the condition of tempo change.

Suppose the tempo accelerates from 60 to 120 over the first 10 beats and 120 to 240 over the next 20. Many functions may be manipulated so that they produce these goal tempi and so that the two passages take the same amount of clock time; but only one continuous function satisfies both these conditions -- the linear function. The slope or increment per beat is 6 (6*10=60) when we move from 60 to 120; it remains 6 (6*20=120) when we move from 120 to 240. A discontinuous equal-ratios function, for example, would be needed to produce these tempi and durations since the equal-ratios function that moves from 60 to 120 over 10 beats would move from 120 to 240 over the next 10 beats, not the next 20.

The "constant duration" property is important in many musical contexts since it allows one to think of proportional tempo change versus proportional note-value change in an intuitively simple manner. The property will be of the first importance with reference to the rhythmic extraction procedures used in Phillip Batstone's music.

5. Second Order Similarities

Closely related to the properties discussed in 2 and 4 above is one in which a "new tempo" emerges from manipulation of an old. In such cases, a new "music-time" score seems to provide a simpler correlation with the performance than the music-time score which generated the performance.

Suppose a series of approximately equal durations were to result from a gradual accelerando of a music time passage which contained gradually increasing note durations. Other things being equal, a listener would probably interpret the passage in an approximately steady tempo. Similarly, if a series of durations were to emerge which could be interpreted, even approximately, in one steady tempo, a listener would most likely make that interpretation.

If a pattern of tempo change is repeated over a set number of beats, this results in, at the very least, a pattern of "beat-duration" repetition. This pattern, even if somewhat irregular, may tend to be heard as "one tempo". This somewhat unusual view of tempo is discussed in detail in connection with musical examples in Sections 8 and 9 of this paper.

3. EQUAL-RATIOS TEMPO CHANGE

In an equal-ratios accelerando or ritard, tempi change is proportionally the same from beat to beat. Consider an equal-ratios accelerando from T1 to T2. As tempi along the equal-ratios curve approach T1, the absolute differences in tempi are smaller and the clock times associated with successive beats approach the clock time associated with the steady tempo, T1. As tempi along the equal-ratios curve approach T2, the absolute differences in tempi are larger and the clock times associated with successive beats approach the clock times associated with the steady tempo, T2.

In general, the larger the T2/T1 ratio, over a set number of beats, the more the function's shape resembles a parabola; the smaller the T2/T1 ratio, the more the function's shape resembles a linear accelerando. The following tables and the graphs included in Example T help make this relationship clear.

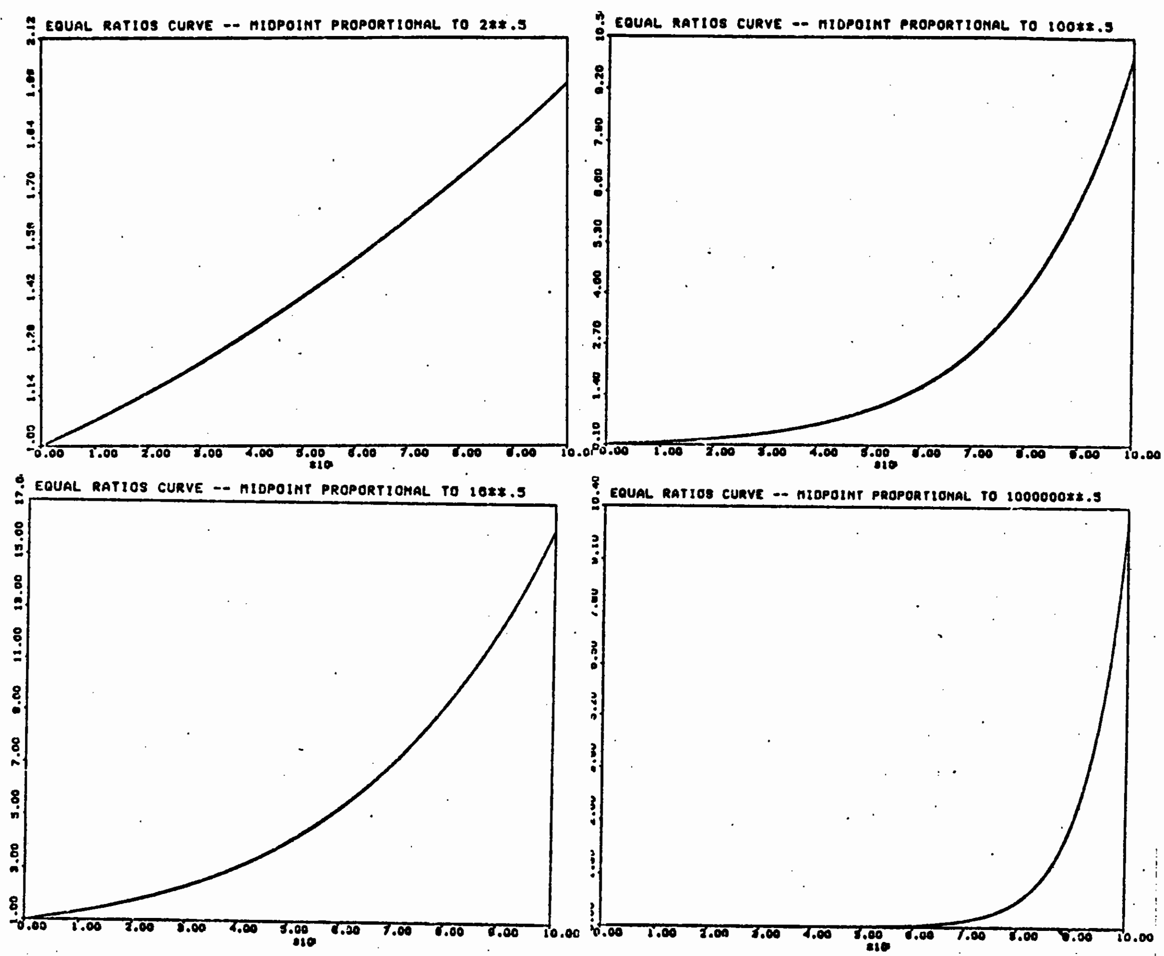
1.	1.5	2.	(linear shape)
1.	1.414	2.	(equal-ratios begin)
1.	2.	4.	
1.	4.	16.	
1.	10.	100.	
1	1000.	1000000.	

The greater the T2/T1 ratio, the less time the accelerando takes and, in that sense, the faster it is. But the greater the T2/T1 ratio, the greater the proportion of the function that stays near T1. In that sense, the greater the ratio, the more slowly one approaches T2.

As the T2/T1 ratio is decreased, we approach the arithmetic mean of the two tempi. To generate functional shapes which will approach T2 more rapidly than the linear function, one can use the inverse equal-ratios function or any of a large number of other functions described in our earlier article.(**1)

In our particular implementation of the equal-ratios curve, we treat ritards as the mirror of accelerandi. Thus, in a ritard, the greater the T2/T1 ratio, the greater the proportion of the function that is near to T1 and, in this sense, the more rapidly it approaches T2.

EXAMPLE I


EQUAL RATIOS CURVE -- MIDPOINT PROPORTIONAL TO 2**.5
EQUAL RATIOS CURVE -- MIDPOINT PROPORTIONAL TO 100**.5
EQUAL RATIOS CURVE -- MIDPOINT PROPORTIONAL TO 16**.5
EQUAL RATIOS CURVE -- MIDPOINT PROPORTIONAL TO 1000000**.5

4. LINEAR TEMPO CHANGE

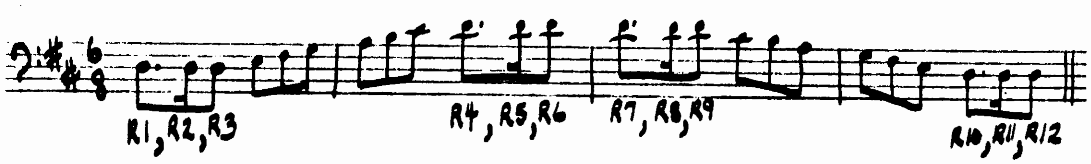
In a linear tempo change, tempi succeed each other by equal differences. An important negative feature of linear tempo change is that all rhythmic proportionality is lost, even that which we earlier called "motivic". In this sense, linear transformations are not intuitively similar to steady tempi since the changes they produce are proportionally unique to each portion of the function. As we observed earlier, perhaps the most important invariant under linear tempo transformation is the preservation of clock time durations of beat segments whose tempi and beat lengths are related by some constant.

Linear accelerandi and ritards are automatically mirror images of each other, thus no convention need be adopted here as was necessary in equal-ratios. Let us consider, then, a linear tempo transformation between T1, the slower tempo, and T2, the faster one. The durational or clock-factor curve associated with this tempi curve would be a hyperbolic decrease from (60/T1) to (60/T2). Suppose we compare these curves with an equal-ratios accelerando from T1 to T2. We see that the tempo change is more rapid than equal-ratios at the beginning of the curve and less rapid at the end. This means that, for any two given tempi, a linear accelerando approaches T2 initially more rapidly than does an equal-ratios accelerando. Conversely, a linear ritard approaches the slower tempo initially less rapidly than does an equal-ratios function. Since the proportion of tempi change is continuously decreasing in a linear accelerando, there is a sense in which a linear accelerando resembles a steady tempo more and more closely. Conversely, in a linear ritard, the proportion of tempi change is continuously increasing and we diverge more and more from a steady tempo. Rapidly changing linear ritards can result in surprising differences in rhythmic proportions between music-time and clock-time scores.

5. LINEAR and EQUAL-RATIOS REALIZATIONS of a SIMPLE PASSAGE

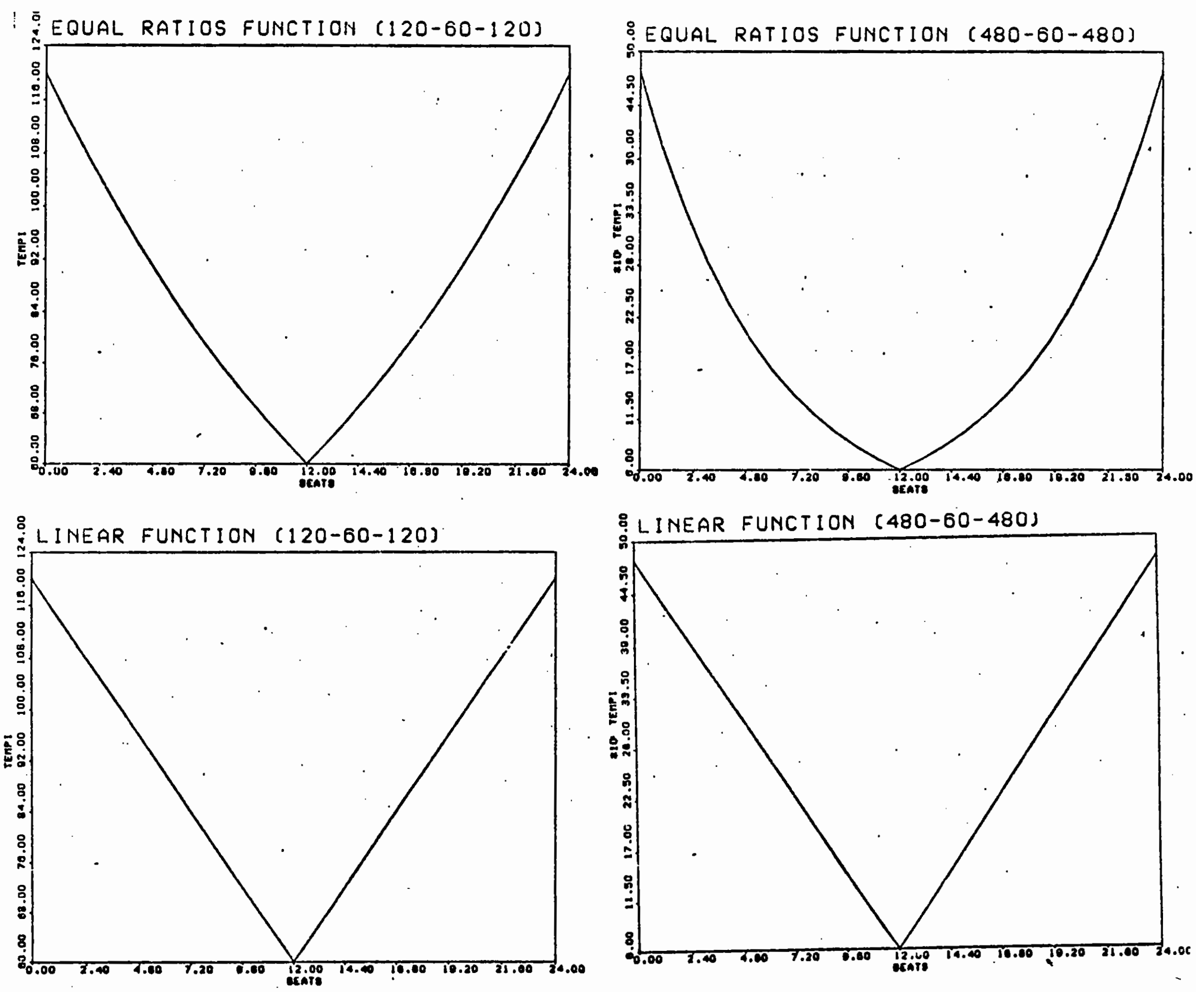
Consider the following written musical example:

<u>MUSIC EXAMPLE 1</u>



We see a passage that ritards as it moves up a D-major scale and accelerates as it moves back down. The same rhythmic motive is repeated as the beginning and end of each ritard and accleration. The motive employs a dotted eighth, sixteenth, and an eighth. The ratios of these in beats are 1.5:.5:1. We will now examine four renditions of this passage. We alternately hear equal-ratios and linear functions applied. The first pair use goal tempi of 120-60-120. The second pair use goal tempi of 480-60-480. The following table compares the ratios of durations produced by each rendition. The graphs of these tempo functions may be seen in Example U.

EXAMPLE U


EQUAL RATIOS FUNCTION (120-60-120)
EQUAL RATIOS FUNCTION (480-60-480)
LINEAR FUNCTION (120-60-120)
LINEAR FUNCTION (480-60-480)
TEMPI
BEATS

6. PHILIP BATSTONE'S PITCH/RHYTHM SETS

Philip Batstone's music makes use of a method of pitch/rhythmic organization in which each particular pitch-class transposition of a pitch-class set is associated with a rhythmic transposition of a set of beat points(**2,**3). Pitch-class transposition is defined in the usual manner as the mod 12 addition of a constant, k, to the original set of pitch-classes, while beat-point transposition is defined as the multiplication of the original beat-point numbers by some constant, L. This means that all intervals between beat-points in the transposed set are L-times as great as the original intervals. Later we will examine in detail the relationships between pitch transposition and rhythmic transposition in this system. We will now consider characteristics of the beat point set itself. We generate beat points for our set from the formula(see **2,pages 69-71):

(b**n)*m,mod z where

b is an integer base
n is an exponent ranging from 0 to c-1
where c is the cardinality of the set
m is a multiplier and in this example
will be set to 1 (see Appendix II)
z is an integer modulus.

We see an example based of powers of 3, mod 35 (b=3,z=35). This modulus has been chosen to produce 12 beat points (c=12).

<u>Powers of 3, mod 35</u>

1 3 9 27 11 33 29 17 16 13 4 12 (beat points in generated order, mod 35.)

Beat points are generated in the order of powers of 3, mod 35 and all powers of 3, mod 35, are generated as beat points. Thus multiplying the beat point numbers by any power of 3, mod 35, will permute the order of the numbers and will generate no new beat points. It is a general rule that if some ordered set of numbers is multiplied by a constant, then the intervals between the numbers in that set are multiplied by the same constant. If we multiply our beat points in generated order by any power of 3, mod 35, we

RITARD

func	T2/T1	R1	R2	R3	R4	R5	R6
st	1/1	1.5	.5	1.	1.5	.5	1.
ER1	2/1	1.35	.48	1.	1.35	.48	1.
LI1	2/1	1.38	.48	1.	1.32	.47	1.
ER2	8/1	1.11	.44	1.	1.11	.44	1.
LI2	8/1	1.29	.47	1.	.83	.37	1.

ACCELERANDO

	T2/T1	R7	R8	R9	R10	R11	R12
ER1	1/2	1.66	.522	1.	1.66	.522	1.
LI1	1/2	1.707	.527	1.	1.62	.516	1.
ER2	1/8	2.035	.569	1.	2.035	.569	1.
LI2	1/8	2.636	.606	1.	1.73	.529	1.

In the ritards, the second and third note of the rhythmic motive become relatively longer in relation to their music-time ratios. Thus the ratio of the first note to the last becomes smaller. Since a ritard contradicts the directionality of the written rhythm (long to short), it is possible that the ritard may modify the sense of the rhythmic relationships. This actually happens at the end of the linear ritard where the dotted eighth's value is shorter than the eighth by the ratio of .83.

In the accelerandi, the second and third notes become relatively shorter in relation to their music-time ratios. Although there is the possibility of a durational inversion between the second and third notes, this does not occur in our examples.

The equal-ratios function produces the same ratios for this motive within any one equal-ratios accelerando or any one equal-ratios ritard. The ratios are, of course, different for different T2/T1 ratios and are different depending on whether the passage is undergoing accelerando or ritard. The linear function produces differing ratios within any ony application of the function.

will simply rotate the set of beat points since they are in increasing powers-of-3 order already. The intervals between the beat points also undergo a simple rotation.

Multiplication of the Generated Set

```
1  3  9 27 11 33 29 17 16 13  4 12   (generated beat points)
 2  6  18 54 162.........            (intervals not reduced
                                      mod 35)
 2  6  18 19 22..........            (intervals, mod 35)

3  9 27 11 33 29 17 16 13  4 12  1   (*3, mod 35 = simple
                                       rotation)
 6 18 54 162...........              (intervals not reduced
                                      mod 35)
 6 18 19 22............              (intervals, mod 35)
```

Suppose we now sort our attack points into numeric order. This forces them all within the span of the modulus, which may be thought of as "one measure". The sum of the intervals will of necessity be equal to the modulus.

Sorted Beat Points and Intervals

```
1  3  4  9 11 12 13 16 17 27 29 33       (beat points)
 2  1  5  2  1  1  3  1  10 2  4  (3)    (intervals)
(2+1+5+2+1+1+3+1+10+2+4+3=35)            (sum of intervals)
```

If we multiply the sorted beat points by L, where L is any power of 3, we produce intervals that are L-multiples of the original sequence. For example, if we multiply the sorted order by 3, we produce intervals that are 3 times the originals. Further, the sum of the intervals is a I-multiple of the original modulus. Thus is we multiply the beat points of the sorted set by 3, we multiply the sum of the intervals by 3 as well.

Multiplication of Beat Points and Intervals

```
3  9 12 27 33  1  4 13 16 11 17 29      (beat points * 3)
 6  3 15  6  3  3  9  3  30 6  12  (9)  (intervals * 3)
(6+3+15+6+3+3+9+3+30+6+12+9=105)        (sum of intervals)
```

If our original set spans J measures, then our L-multiple of the original set will span J*L measures. The "closest" multiple to the original set occurs when L=b, the number originally raised to a power. With reference to our earlier example the closest multiple is 3. In a sense there is no upper limit to this procedure. If we accept the modulus as a limit for multiplication, then the farthest removed multiplier for our example would be 3**5, mod 35. This number would multiply the original durations by 33, span 1155 beats, and would take 33 times as long to play as the original.

Since our rhythmic set is of size 12, we may associate with its elements a pitch-class set of size 12. There are optimal ways of making this association. Consider the rhythmic set in its generated order. Let us associate with this set a set of pitch-classes whose order is determined by some complete pitch-class cycle of the twelve-tone system. Complete pitch-class cycles are formed by those pitch-class-intervals which are relatively prime to 12, namely 1,11,5,and 7. We show an example based on the "chromatic scale", the cycle generated by pitch-class-interval 1.

<u>Beat-point and Pitch-class Association</u>

1	3	9	27	11	33	29	17	16	13	4	12	(generated beat points)
0	1	2	3	4	5	6	7	8	9	10	11	(generated pitch classes)
1	3	4	9	11	12	13	16	17	27	29	33	(sorted beat points)
0	1	10	2	4	11	9	8	7	3	6	5	("sorted" pitch classes)

These pc numbers may be thought of as "order numbers" of the generated beat-point set. The shuffled order shows the order positions in the original generated set of the sorted set. Suppose some constant, k, is added, mod 12, to the set of order numbers. This new set of order numbers shows the positions in the generated beat-point set of a second beat-point set poduced by multiplying the sorted beat-point set by b**n, where b is the integer base and n is an exponent between 1 and 11. Since we think of transposition as the addition, mod 12, of some constant, k, to a set of pitch class numbers, we have here associated each possible pitch class transposition of a pitch class set with a multiplication of a rhythmic set.

In order to see why this is so, it is helpful to consider the following two strings of a base number raised to a power:

Addition of a Constant to a String of Exponents

```
3**0  3**3  3**1  =  1, 27, 3;  intervals 26,-24
   (add 1 to exponents)
3**1  3**4  3**2  =  3, 81, 9;  intervals 78,-72
```

We see that by adding 1 to our string of exponents we multiply all numbers and intervals by 3. For the general case, adding a constant, n, to all exponents in a string will multiply all numbers and intervals by b**n, where b is our integer base. The numbers and intervals between numbers so generated may be reduced mod z. For most of our applications, the reduction mod z will not be useful. This reduction would destroy the multiplicative relationship between our various simultaneous rhythmic statements. Finally consider our original pitch class numbers as exponents for the integer base, b, where b = 3.

PC's as Beat-point Exponents

	1	3	4	9	11	12	13	16	17	27	29	33	(values)
	0	1	10	2	4	11	9	8	7	3	6	5	(exponents)
3**^													(base)

Adding 1 to all the exponents must, of necessity, multiply all beat-point differences, that is all durations, by b, the base. For the general case, adding some constant, n, to all the pitch class numbers-- thereby producing a pitch class transposition of the original pitch class set -- will produce a rhythmic statement whose rhythmic intervals are b**n times as great as the original sorted rhythmic statement. This also means that one statement of the original set will take up the same amount of music-time as n statements of the original set at a pitch-class transposition of n.

<u>(Exponents + 1) Triple Beat-point values and Durations</u>

```
     3  9 12 27 33  1  4 13 16 11 17 29 (values)
     1  2 11  3  5  0 10  9  8  4  7  6 (exponents)
3**^                                    (base)
```

The following example shows two vertically aligned statements of our pitch/rhythm sets. The relative pitch transposition levels are 0 and 1 and thus the relative tempo relations are 3:1. One original set moves at three times the tempo of the transposed set and thus one transposed set takes up three times as much music-time as three untransposed sets. Voice 1 makes three statements of the untransposed set while voice two makes one statement at t=1. The beat-point intervals (bpi's) -- the durations -- of the two voices are given as the second line of each voice. The third line shows the pitch classes (pc's) of each voice.

<u>Two Simultaneous Statements at n=0, n=1</u>

```
voice a  1  3  4  9 11 12 13 16 17 27 29 33 (bp's)
 "   "    2  1  5  2  1  1  3  1 10  2  4  3 (bpi's)
 "   "   0  1 10  2  4 11  9  8  7  3  6  5 (pc's)

voice b     3     9    12          27    33 (bp's)
 "   "         6     3       15       6     3 (bpi's)
 "   "      1     2    11           3     5 (pc's)

**********

voice a  1  3  4  9 11 12 13 16 17 27 29 33
 "   "    2  1  5  2  1  1  3  1 10  2  4  3
 "   "   0  1 10  2  4 11  9  8  7  3  6  5

voice b  1     4          13 16
 "   "     3     9          3     30
 "   "   0    10           9  8

**********

voice a  1  3  4  9 11 12 13 16 17 27 29 33
 "   "    2  1  5  2  1  1  3  1 10  2  4  3
 "   "   0  1 10  2  4 11  9  8  7  3  6  5

voice b              11          17    29
 "   "                     6        12     (9)
 "   "                4           7     6
```

We mentioned earlier that our pitch-class set could be formed from any cycle that is relatively prime to 12. In order to square this idea with the "exponential" interpretation of the pc numbers, it is sufficient to introduce the idea that the set of pc's may be multiplied by any constant, m, where m is one of the numbers relatively prime to 12. For the general case m must be relatively prime to l, where l is the cardinality of the sets. The following shows our above example under pc multiplication by 5, mod 12.

<u>Two Simultaneous Statements, PC's * 5</u>

```
voice a    1   3   4   9  11  12  13  16  17  27  29  33  (bp's)
  "   "      2   1   5   2   1   1   3   1  10   2   4   3 (bpi's)
  "   "    0   5   2  10   8   7   9   4  11   3   6   1  (pc's)

voice b        3       9      12              27      33  (bp's)
  "   "              6       3          15          6       3  (bpi's)
  "   "        5      10       7               3       1  (pc's)

**********

voice a    1   3   4   9  11  12  13  16  17  27  29  33
  "   "      2   1   5   2   1   1   3   1  10   2   4   3
  "   "    0   5   2  10   8   7   9   4  11   3   6   1

voice b    1       4              13  16
  "   "      3       9                3      30
  "   "    0       2               9   4

**********

voice a    1   3   4   9  11  12  13  16  17  27  29  33
  "   "      2   1   5   2   1   1   3   1  10   2   4   3
  "   "    0   5   2  10   8   7   9   4  11   3   6   1

voice b                   11              17      29
  "   "                        6              12      (9)
  "   "                    8              11       6
```

We should now be able to understand more fully the close relationship between the pitch-class and beat-point operations of this system. A pitch-class is, by definition, simply one part of an oddly formed exponent -- pc 6 needs a registral number to become a frequency determinant. If we give it the register 8, we produce the octave.pc number, 8.06. This is an oddly formed exponent in the sense that

.06 should be converted to .5(6/12) before the exponent is associated with its normal base, 2. Since we have just shown how pc numbers may be regarded as beat-point exponents, this means that the same numbers are serving as both rhythmic and pitch exponents.

Each statement of the basic set or one of its transpositions may be spoken of as a "set instance". It is often convenient to think of the "faster moving" sets of the above combinations as the "background". The "slower moving" sets are extracted from this background and may be spoken of as "extracted sequences". One straight-forward interpretation of this kind of rhythmic scheme would have "slower and slower" extracted voices against a steady background. Another interpretation would allow each set instance to state the same durations against an appropriately adjusted, even faster moving, background. Both these interpretations involve different tempi and may be easily extended to involve changing tempi.

7. MENSURATION CANONS and TEMPO CHANGE

In an earlier discussion, we commented that tempi change destroys rhythmic proportionality in its simplest sense. In a steady tempo there are four quarters per whole note and the quarters proceed at four times the speed of the wholes. Consider the following passage played at a steady tempo.

MUSIC EXAMPLE 2

The following equations are true for steady tempo:

$b/a=d/c$ since all quarter notes are equal.
$b/a=j/i$ since the ratio of all "equal note values" is 1.

In an accelerando the durational values of the quarter notes become progressively shorter: similarly, the second whole note is shorter than the first. Thus

$a/x > 1$, where $x = b,c,d,e,f,g,h$
$i/j > 1$.

If we consider an equal-ratios accelerando, then

$$b/a = d/c = f/e = h/g$$
$$(a+b)/(b+c)=a/b$$
$$(a+b+c)/(b+c+d)=a/b$$
$(a+b)/(c+d)$ does not equal a/b but
$$(a+b)/(c+d) = (a/b)^{2}$$
$(a+b+c+d)/(e+f+g+h)$ does not equal (a/b)
$$(a+b+c+d)/(e+f+g+h) = i/j$$
thus (a/b) does not equal (i/j)
but (i/j) does equal $(a/b)^{4}$.

The above equations make clear that, in equal-ratios accelerandi, proportional rhythmic relations among simultaneous voices are lost. An "ideal" function to avoid this problem would be one in which proportionality was maintained over all possible comparisons. Thus

$$(b/a) = (d/c) = (j/i)$$

Unfortunately, such a tempo-change function contains a logical contradiction and cannot exist; only the steady tempo function fulfills this condition.

These observations are germane to our Batstone example since the identification of rhythmic proportion in Batstone-type mensuration canons may be of perceptual importance. If we apply an accelerando to our earlier "schematic", two-voice example, we can forecast that the durational proportions of the faster voice will no longer be the same as those of the slower. Suppose our acceleradno used an equal-rations function and moved from MM 100 to 300 over the 35 beats of the slower voice. (See Section 8 and Appendix III as well.) The following table summarizes the results of this accelerando by comparing the music-time proportions with the actual resulting durations for the first five notes of each voice:

Equal-Ratios Accelerando

props	2	1	5	2	1
v 2 durs	1.127	.5376	2.4488	.87686	.41822
v 2 ratios	2.695	1.285	5.855	2.0967	1.
v 1 durs	.39172	.19281	.93436	.36027	.17733
v 1 ratios	2.209	1.0873	5.2691	2.0317	1.

A linear accelerando produces the following results:

Linear Accelerando

props	2	1	5	2	1
v 2 durs	1.078	.5	2.1955	.764	.3621
v 2 ratios	2.977	1.381	6.063	2.1099	1.
v 1 durs	.38536	.1875	.89037	.33603	.16407
v 1 ratios	2.349	1.1429	5.4269	2.0481	1.

Neither of these curves, nor any other curve, allows each voice to present the same proportional relationship. This is an important limitation on the coherence of mensuration canons undergoing tempo change. Indeed, it suggests that other rhythmic features may be more important compositional resources of Batstone-type methods than the fact that the voices are, in some sense, of the same proportional rhythm.

8. SIMULTANEOUS LINEAR ACCELERANDOS

We will now construct a musical example based on the Batstone sets listed on page 20 of Section 6. The example will have the following characteristics:

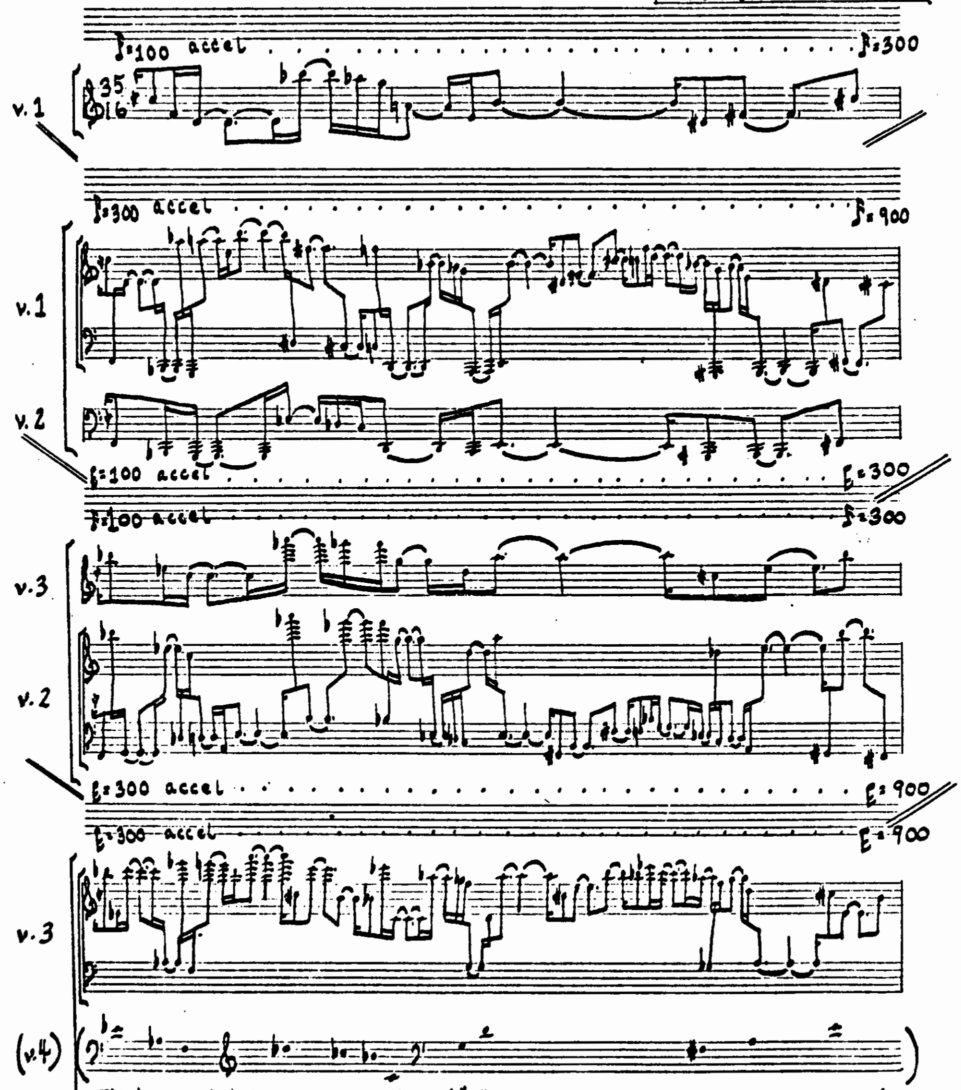
1. three voices will be employed.
2. each voice will enter at a pitch-class transposition of +5 in relation to the immediately preceding voice.
3. the pitch-class levels of the entries will thus be 0,5,10.
4. each voice will make four statements of its pitch rhythm set.
5. the last three statements of each voice will take the same amount of time as the first statement of that voice.
6. the first statement of each voice will take the same amount of time.
7. each new voice will enter after one complete statement of the preceding voice.
8. each new voice will begin at the same time as the second statement of the voice already playing and will take up the same amount of time as three statements in that previous voice.
9. each voice will begin at the same tempo and accelerate to three times that tempo over its first statement.
10. each voice will accelerate to nine times the original tempo (or three times the new) over its final three statements.
11. one continuous linear function will be used to control the shape of accelerando for each voice.
12. voice 1 will begin in register 9 (on note 9.00); voice 2 will begin in register 6 (on note 6.05); and voice 3 will begin in register 9 (on note 9.10).
13. registration will be used to help make the voices distinct with one exception. The "slow moving", extracted voice will always be doubled at the unison rather than the octave, when such duplications arise between the "relatively background" voice and the extracted voice.

A taped example was played here. See Appendix III for Pass2 Printout of that example. See Music Example 3 for a "notated version".

MUSIC EXAMPLE 3

Mensuration Canon

Dynamic markings and articulation indications are not included in this score.



This line is stated registrally and in terms of "attack point rhythm". It is not articulated in terms of durations and doublings.

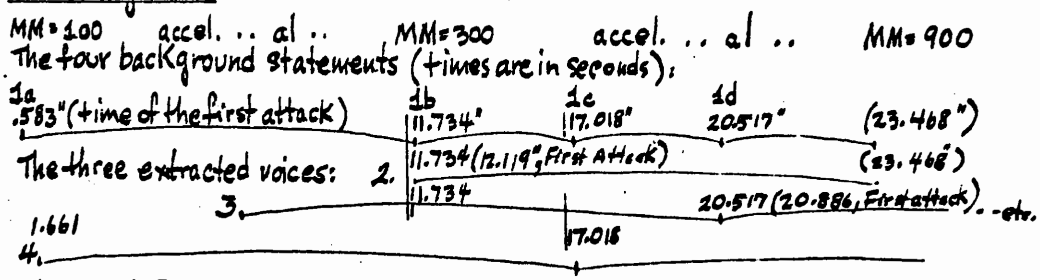
We note that:

1. the use of the linear function has brought about the rhythmic alignment of the voices through the use of one continuous function.
2. in order to make foreground statements (extracted sequences) take the same amount of time, it was necessary for the background to be continuously accelerating. In fact, the original "background voice" disappears with the entry of voice 3. If it had continued as a "real line", it would have eventually reached the tempo of 5100 beats per minute.
3. the durations (actually "attack point differences) of each "extracted voice" are the same as those of the original voice. From this fact, it is obvious that each "extracted voice" takes the same amount of time as the original and that each "bar" takes the same amount of total clock time. The temporal relations of "one bar to another" are thus exactly the same, and, in a special sense, each entry is in the "same tempo". (See Section 9).
4. the proportions revealed by any two simultaneous statements are not and cannot be the same.
5. the proportional differences in rhythmic values are less as the passages accelerate since a linear accelerando approaches, in this sense, a straight line (see Section 4).

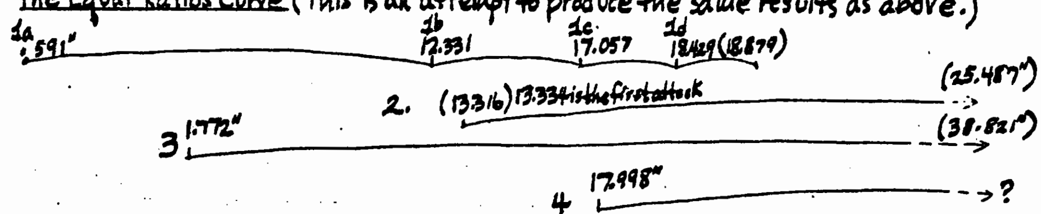
Example V

This example shows a three-voice extraction from four consecutive statements of a set undergoing acceleration. It is shown twice: once using the straight-line curve and once using the equal-ratios curve. (MM = the number beats/minute)

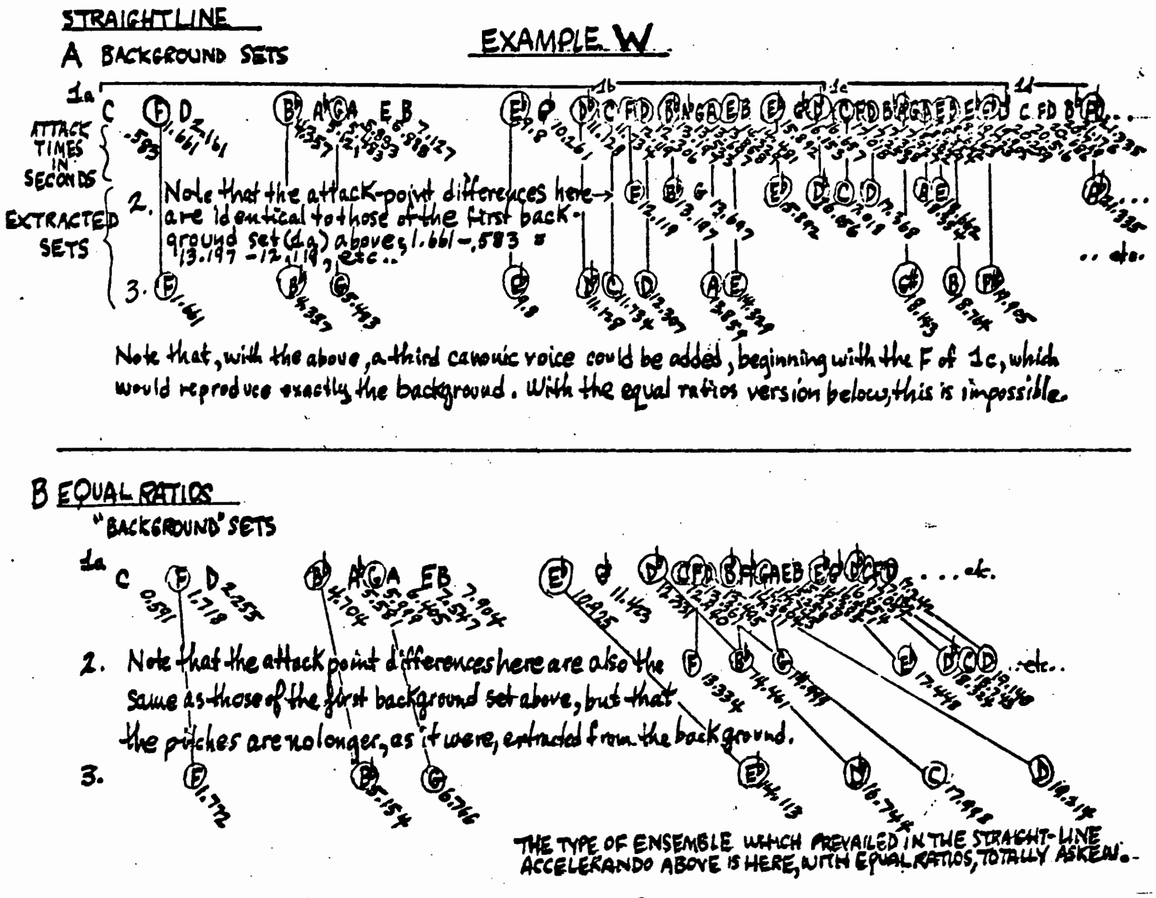
The Straight Line



The Equal-Ratios Curve (This is an attempt to produce the same results as above.)



Notice that here the times do not line up at all. See example W for a detail.

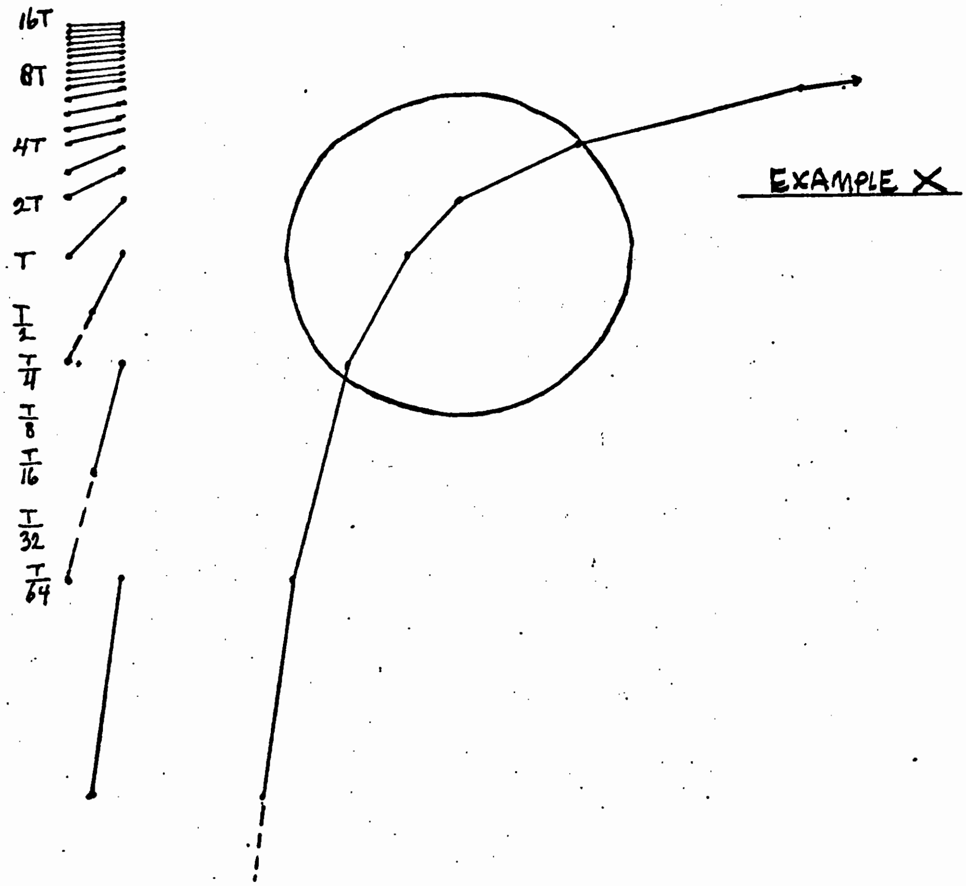

STRAIGHT LINE
EXAMPLE W
A BACKGROUND SETS
ATTACK TIMES IN SECONDS
EXTRACTED SETS
Note that the attack-point differences here are identical to those of the first background set (1a) above; 1.661 − .583 = 13.197 − 12.119, etc..
Note that, with the above, a third canonic voice could be added, beginning with the F of 1c, which would reproduce exactly the background. With the equal ratios version below, this is impossible.
B EQUAL RATIOS
"BACKGROUND" SETS
Note that the attack point differences here are also the same as those of the first background set above, but that the pitches are no longer, as it were, extracted from the background.
THE TYPE OF ENSEMBLE WHICH PREVAILED IN THE STRAIGHT-LINE ACCELERANDO ABOVE IS HERE, WITH EQUAL RATIOS, TOTALLY ASKEW.

9. TEMPO CHANGE as STEADY-STATE (P. Batstone)

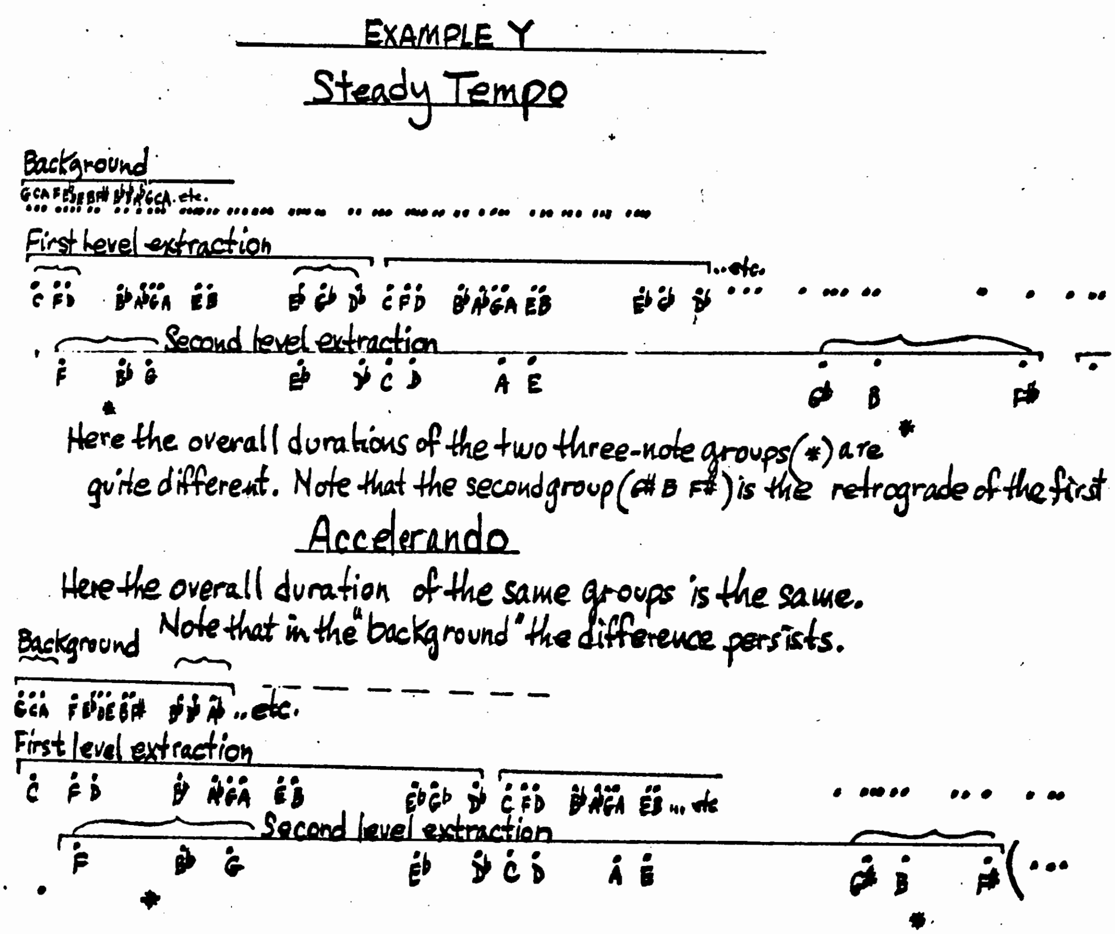
The preceding musical example by Prof. Rogers uses the same pitch/rhythm set as the one discussed in my Examples V and W. From these examples and from the preceding musical one, we see again that the use of the equal ratios curve preserves proportionality according to the tempo change, while extracted sequences which are meant to duplicate previous rhythmic relationships do not and cannot. On the other hand, use of the straight line function, while of necessity altering proportionality of successive statements, does so in order that extracted sequences can duplicate exactly the rhythms of set instances which occurred previously at the <u>same</u> tempo.

As we will see below, this fact is not the only reason for my choice of the linear function and the associated hyperbolic clock factor as part of my compositional methods. But this fact allows me to think of any passage being composed as presenting one particular primary tempo of my own choosing and not any one of a number of simultaneous tempi.

Most of my compositions contain passages which contain from three to five voices, each undergoing tempo change. It is at the medium tempo that the most imitation occurs. If we take a situation where, for instance, the tempo change is from Tempo 1 (T) to twice that tempo (2T) over the statement of one set instance, and from that new tempo to twice again over the next <u>two</u> statements (ending at 4T), it is clear that the relative amount of tempo change over the third statement is less than that over the first. In short, the faster the general relative tempo, the slower the rate of change. Conversely, the slower the general tempo relatively, the greater the rate of change. The next slower statement, the one which would have preceded statement number 1 on the same tempo function, would double its tempo over the latter half. In Example X, I have shown on the left the relative tempo change of various sets and set segments according to relative general tempo and according to the curve just described. On the right I have gathered these together, not exactly against time, but simply on a curve illustrating the differences in rate of change between such statements.



Note that the medium tempi, those within the circle, are those among which there is the greatest variety in proportionality. Please keep in mind that for my purposes this variety is desirable as well as necessary to maintain tempo identity. One might say that this variety is the overriding reason for using tempo change when it is not perceived in the traditional way, eg., as stringendo, rallentendo, etc.. In a passage to which the above tempo relationships apply, it is those statements which move from T to 2T which present what I call the "basic tempo" of the passage. I call this the basic tempo even though tempo change is involved, because the tempo change is not perceived as such. In fact, the choice of the degree of tempo change is often dictated by concerns of identity of rhythmic figures <u>within</u> set instances (and not just between them). This is the kind of redundancy which is characteristic of steady tempo and, as can be seen from Example Y, tempo change is necessary in order to create this identity. It is in this sense that I speak of "tempo change as steady state".


EXAMPLE Y
Steady Tempo
Background
First level extraction
Second level extraction
Here the overall durations of the two three-note groups (*) are quite different. Note that the second group (C# B F#) is the retrograde of the first
Accelerando
Here the overall duration of the same groups is the same.
Note that in the "background" the difference persists.
Background
First level extraction
Second level extraction

Now the basic tempo, however fast or slow it may be arbitrarily, can be altered in the usual way in the performance completely independently of the considerations discussed here. In fact, there is nothing in this discussion which precludes the application of an expressive ritard or stringendo here and there, even if it appears to contradict the tempo curve being used.

The distinction is between two different uses of tempo change. One is really on the performance level, while the other is entirely structural, having to do with harmonic ensemble as well as motivic identity. As shown in Example W, the rhythmic relationship of the attack points of several simultaneous voices is the first and foremost reason for the preference for one kind of tempo curve over another.

To sum up, there are three advantages to the choice of the hyperbolic clock factor which make it my choice in most of my recent music. They are:

1. the preservation of harmonic contexts,
2. the identifiability of melodic material which recurs, and
3. the singularity of tempo at any given time.

The latter, of course, not only is achieved at the expense of proportionality between statements at different tempi, but, in fact, <u>depends</u> on it.

Since what is true of combinations like that of Example W also holds true for combinations of combinations, there are a great many possibilities of combination, including those of diminution (the addition of voices to a given passage) as well as reduction (the removal of voices from a given passage). There are also, of course, other moduli than 35, the modulus of the above examples (For a longer musical example using the above set see **4). Unitl recently, all the moduli I have used have been factors of (2**12)-1 or of (3**12)-1(**2,**3). The exponent is 12 because the group of pitch classes is of order 12.

For several years, I have been using the computer to realize various studies which employ 53-tone equal temperament. Aside from the nearly perfect harmonic "fifths" and "thirds", which is well known, there is the fact that 53 is a prime number. This means that any interval within the group can generate the entire group.

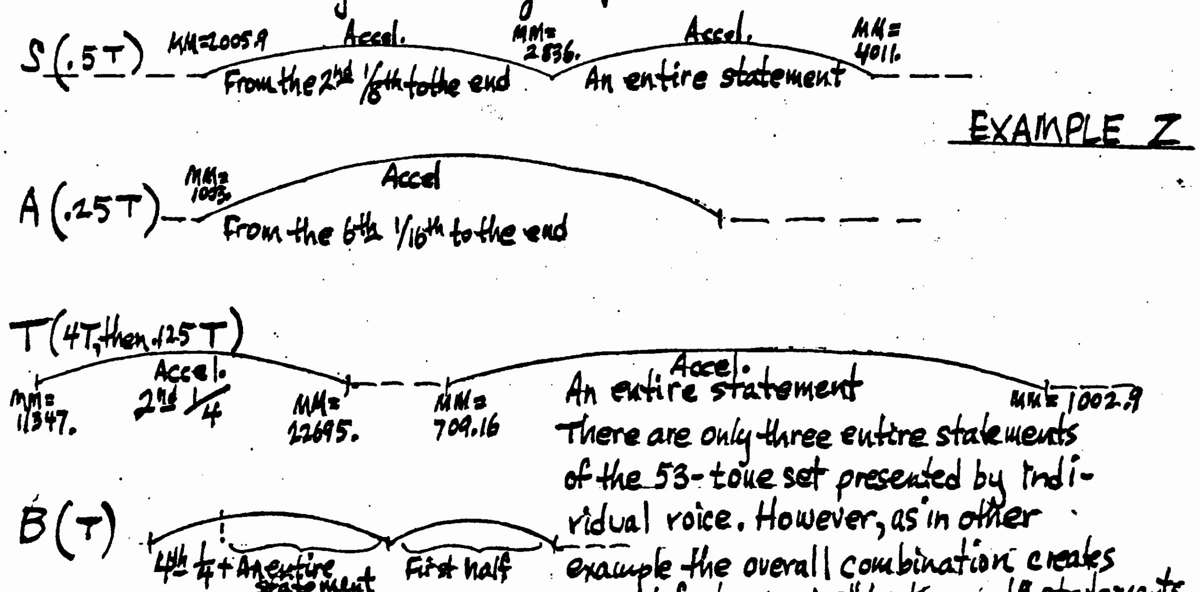
Therefore the number of possible sets (excluding transpositions but not retrogrades or inversions) for the use of my techniques using powers of 2 is not (2**53-1)/53, but (2**53-1), a large number indeed. For convenience, I use as my modulus the number 6361, one of the prime factors of 2**53-1. (See Appendix II.)

Example Z shows the disposition of the four voices of the taped example you will hear. This is one of the four "Quartets" from the Ninth Study of <u>Nine Studies for Computer</u>, which were completed in 1978. This example was realized on the DEC10 computer at the University of New Hampshire using MUSIC4BF and a set of special PASS1 tempo and note-generation routines.

(Taped example was played.)

While my pitch-rhythm techniques do not necessitate computer realization, the complexities involved in such 53-tone studies, particularly when curves of tempo change are applied, make such composition without the computer almost insurmountably difficult.

A visual aid showing the disposition of the four voices in the recorded example.*
The relative tempo relationships are shown at left (.5T, .25T, etc.)
Some of the starting and ending tempi are also shown.



*The fifth quartet from the last of "<u>Nine Studies for Computer</u>"

BATSTONE MODULI

Batstone rhythmic sets are generated from the formula:

(b**n)*m,mod z

where

b is the base number
m is a multiplier
z is the modulus and
n ranges from 0 to c-1 where
c is the length of (number of elements in) the set.

We may make the following observations:

1. the largest modulus is (b**c)-1. We will call this particular modulus z1.
2. other and smaller z's may be found providing:
   1. the particular modulus, z, is larger than c, the size of the set.
   2. the particular modulus, z, is a factor of z1.
   3. in the formula z=(b**m)-1, m is not a factor of c.
   4. the new z is not a factor of a larger "new z" that fails because of criteria 3 above.

Thus for sets of size 12, base 2:

1. 2**12-1 = 4095 is z1, the largest modulus.
2. the following factors of z1 will work as new z's:

   13,35,39,45,65,91,105,117,195
   273,315,455,585,819,1365

3. other factors of 4095 will not produce enough beat points to work as new z's since they violate one of the criteria listed above. For example:
   1. 63 will not work since (2**6)-1 =63 and 6 is a factor of 12.
   2. 15 will not work since (2**4)-1 =15 and 4 is a factor of 12.
   3. 21 will not work since 21 is a factor of 63 and 63 will not work.

PATSTONE MULTIPLIERS

We may make the following observations about the multipliers which may be used;

1. No multiplier can produce a set of lager cardinality than that of the original set.

2. If the multiplier is included in the original series, we simply permute the original series and no new beat points are created.

3. If the multiplier is not in the original set and is relatively prime to the modulus, a new set of the same cardinality as the original will be produced.

4. If the multiplier is a factor of the modulus then the maximum cardinality of the set it will produce is S1, where S1=S-1 and S=(modulus/multiplier) provided S1 is no larger than the size of the original set. The actual cardinality of the set depends on the number of numbers in the original set which are distinct mod S, and thus the actual size of the set may be less than S1.

5. If the multiplier is included in a set produced by one of the above operations, then multiplying the original set by the multiplier will produce a permutation of the set in which the multiplier is included.

6. Sets produced by multiplication are either totally identical in content or totally distinct, provided the base and modulus remain the same. They cannot share some numbers and not share others.

7. All possible numbers up to and including the modulus may be generated by multiplying the set by appropriate multipliers.

Two examples of this procedure follow:

Powers of 2, mod 35

1	2	4	8	16	32	29	23	11	22	9	18	(*1)
3	6	12	24	27	33	13	19	31	34	17	26	(*3)
5	10	20	5	10	20	etc.............						(*5)
7	14	28	21	7	14	28	21	etc.......				(*7)
15	30	25	15	15	30	25	15	etc.......				(*15)
0	0	etc........................										(*35)

If we try all multipliers from 0-35, mod 35, we may make the following observations regarding the above example.

1. multiplying by any power of 2, mod 35, yields a permutation of the numbers in the original set.

2. multiplying by 3 yields a new set of size 12 since 3 is relatively prime to 35 and is not included in the original set.

3. multiplying the original set by any number found in the "m=3" set results in a set that is a permutation of the "m=3" set.

4. multiplying by 5 can produce, at maximum, a new set of cardinality 6 since (35/5)-1 is 6. Since there are only 3 numbers distinct, mod 7, in the original set, multiplying by 5 will result in a new set of cardinality 3.

5. multiplying by 7 can produce, at maximum, a set of cardinality 4 since (35/7)-1 is 4. Since there are four numbers in the original set which are distinct, mod 5, a new set of cardinality 4 is produced by this operation.

6. if the original set is multiplied by 3, the "missing" three numbers which are distinct, mod 7, are produced. Multiplying this set by 5 will produce the other numbers which could have been found in the "m=5" set. Multiplication by 3 and then by 5 is equivalent to multiplying the original set by 15.

<u>Powers of 3, mod 35</u>

1	3	9	27	11	33	29	17	16	13	4	12	(*1)
2	6	18	19	22	31	23	34	32	26	8	24	(*2)
5	15	10	30	20	25	5	15	10	30	20	25	(*5)
7	21	28	14	7	21	28	14	7	21	28	14	(*7)
0	0	etc...										(*35)

1. multiplication by any power of 3, mod 35, will simply permute the numbers in the original series.

2. multiplication by 2 will produce a new set of size 12 since 2 is relatively prime to 35 and is not in the original set.

3. multiplication by 5 will produce a new set of cardinality 6 since there are 6 numbers which are distinct, mod 7, in the original set.

4. multiplication by 7 will produce a new set of size 4 since there are 4 numbers which are distinct mod 5 in the original set.

Finally, consider a set based on powers of 2, mod 4095, the largest modulus. Suppose the resulting set were multiplied by 65, mod 4095. In order for there to be 12 elements in the resulting set, there must be 12 numbers distinct, mod 63, in the original set (4095/65=63.). But there are only 6 numbers, and we produce a new and smaller set even though we are multiplying by a number which is itself a valid modulus. If we were to multiply by 63, however, we would produce a set of size 12 since there are 12 numbers which are distinct, mod 65, in the original set.

PASS2 PRINTOUT of MUSIC EXAMPLE 3

VOICE 1 BEGINS AND STATES ITS DURATIONS.

		st time	dur	oct.pc	
I	67.	0.583	1.078	9.00	
I	67.	1.661	0.500	8.05	
I	67.	2.161	2.195	8.02	
I	67.	4.357	0.764	9.10	
I	67.	5.121	0.362	9.08	
I	67.	5.483	0.350	9.07	
I	67.	5.833	0.985	8.09	
I	67.	6.818	0.309	8.04	
I	67.	7.127	2.673	8.11	
I	67.	9.800	0.462	8.03	
I	67.	10.261	0.866	8.06	
I	67.	11.128	0.408	9.01	
I	67.	11.734	0.385	9.00	(V 1, 2ND STATEMENT)
I	67.	12.119	0.188	6.05	

SECOND VOICE REPEATS THE ORIGINAL DURATIONS OF THE FIRST VOICE WHILE THE FIRST VOICE CONTINUES.

I	68.	12.119	1.078**	6.05	(V 2 BEGINS)
I	67.	12.306	0.890	9.02	
I	67.	13.197	0.336	5.10	
I	68.	13.197	0.500**	5.10	
I	67.	13.533	0.164	9.08	
I	67.	13.697	0.162	5.07	
I	68.	13.697	2.195**	5.07	
I	67.	13.858	0.470	9.09	
I	67.	14.329	0.152	9.04	
I	67.	14.481	1.412	9.11	
I	67.	15.892	0.261	7.03	
I	68.	15.892	0.764**	7.03	
I	67.	16.153	0.503	9.06	
I	67.	16.656	0.243	7.01	
I	68.	16.656	0.362 (etc)	7.01	
I	67.	17.018	0.235	7.00	
I	68.	17.018	0.350	7.00	
I	67.	17.253	0.115	9.05	
I	67.	17.368	0.559	6.02	
I	68.	17.368	0.985	6.02	
I	67.	17.927	0.215	8.10	
I	67.	18.142	0.106	8.08	
I	67.	18.249	0.105	8.07	
I	67.	18.354	0.309	5.09	
I	68.	18.354	0.309	5.09	
I	67.	18.662	0.101	6.04	
I	68.	18.662	2.673	6.04	
I	67.	18.763	0.960	8.11	
I	67.	19.723	0.182	8.03	
I	67.	19.905	0.354	8.06	
I	67.	20.259	0.173	9.01	

I	67.	20.517	0.169	9.00	
I	67.	20.686	0.083	9.05	
I	67.	20.769	0.407	9.02	
I	67.	21.176	0.158	8.10	
I	67.	21.335	0.078	5.08	
I	68.	21.335	0.462	5.08	
I	67.	21.413	0.078	8.07	
I	67.	21.491	0.230	8.09	
I	67.	21.721	0.076	8.04	
I	67.	21.797	0.727	5.11	
I	68.	21.797	0.866	5.11	
I	67.	22.524	0.140	8.03	
I	67.	22.663	0.408	6.06	
I	68.	22.663	0.274	6.06	
I	68.	22.937	0.134	8.01	(LAST NOTE, V 1)
I	67.	23.269	0.385	6.05	(V 2, 2ND STATEMENT)
I	67.	23.654	0.188	9.10	

THIRD VOICE REPEATS THE DURATIONS OF THE FIRST VOICE
WHILE THE SECOND VOICE CONTINUES.
NOTE THAT THIS ENSEMBLE PASSAGE IS, THE SAME AS THE
SECOND ENSEMBLE PASSAGE.

I	68.	23.654	1.078**	9.10	(V 3 BEGINS)
I	67.	23.842	0.890	6.07	
I	67.	24.732	0.336	9.03	
I	68.	24.732	0.500**	9.03	
I	67.	25.068	0.164	7.01	
I	67.	25.232	0.162	9.00	
I	68.	25.232	2.195**	9.00	
I	67.	25.394	0.470	7.02	
I	67.	25.864	0.152	6.09	
I	67.	26.016	1.412	7.04	
I	67.	27.428	0.261	10.08	
I	68.	27.428	0.764**	10.08	
I	67.	27.689	0.503	7.11	
I	67.	28.192	0.243	10.06	
I	68.	28.192	.362 (etc)	10.06	
I	67.	28.554	0.235	10.05	
I	68.	28.554	0.350	10.05	
I	67.	28.788	0.115	7.10	
I	67.	28.904	0.559	9.07	
I	68.	28.904	0.985	9.07	
I	67.	29.463	0.215	7.03	
I	67.	29.678	0.106	7.01	
I	67.	29.784	0.105	7.00	
I	67.	29.889	0.309	9.02	
I	68.	29.889	0.309	9.02	
I	67.	30.198	0.101	9.09	
I	68.	30.198	2.673	9.09	
I	67.	30.299	0.960	7.04	
I	67.	31.258	0.182	6.08	
I	67.	31.440	0.354	6.11	
I	67.	31.795	0.173	7.06	
I	67.	32.053	0.169	7.05	
I	67.	32.222	0.083	7.10	

I	67.	32.305	0.407	7.07	
I	67.	32.712	0.158	7.03	
I	67.	32.870	0.078	9.01	
I	68.	32.870	0.462	9.01	
I	67.	32.949	0.078	7.00	
I	67.	33.027	0.230	7.02	
I	67.	33.256	0.076	6.09	
I	67.	33.332	0.727	9.04	
I	68.	33.332	0.866	9.04	
I	67.	34.059	0.140	6.08	
I	67.	34.198	0.274	9.11	
I	68.	34.198	0.408	9.11	
I	67.	34.472	0.134	6.06	(LAST NOTE, V 2)
I	67.	34.804	0.385	9.10	(V 3, 2ND STATEMENT)
I	67.	35.190	0.188	8.03	

THIRD ENSEMBLE PASSAGE ENDS AS SECOND VOICE DROPS OUT.
THIRD VOICE CONTINUES WITH ITS LAST THREE STATEMENTS.

I	67.	35.377	0.890	10.00	
I	67.	36.268	0.336	7.08	
I	67.	36.604	0.164	10.06	
I	67.	36.768	0.162	7.05	
I	67.	36.929	0.470	10.07	
I	67.	37.400	0.152	10.02	
I	67.	37.552	1.412	10.09	
I	67.	38.963	0.261	9.01	
I	67.	39.224	0.503	10.04	
I	67.	39.727	0.243	8.11	
I	67.	40.089	0.235	8.10	
I	67.	40.324	0.115	10.03	
I	67.	40.439	0.559	8.00	
I	67.	40.998	0.215	9.08	
I	67.	41.213	0.106	9.06	
I	67.	41.319	0.105	9.05	
I	67.	41.424	0.309	7.07	
I	67.	41.733	0.101	8.02	
I	67.	41.834	0.960	9.09	
I	67.	42.794	0.182	9.01	
I	67.	42.976	0.354	9.04	
I	67.	43.330	0.173	9.11	
I	67.	43.588	0.169	9.10	
I	67.	43.757	0.083	10.03	
I	67.	43.840	0.407	10.00	
I	67.	44.247	0.158	9.08	
I	67.	44.406	0.078	7.06	
I	67.	44.484	0.078	9.05	
I	67.	44.562	0.230	9.07	
I	67.	44.792	0.076	9.02	
I	67.	44.867	0.727	7.09	
I	67.	45.594	0.140	9.01	
I	67.	45.734	0.274	8.04	
I	67.	46.008	0.134	8.11	(LAST NOTE, V3)

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3. continued in Perspectives of New Music, Fall-Winter 1972, pages 92-111.

4. Phoenix and Turtle, a setting of William Shakespeare's poem for live soprano and electronic tape by Philip Batstone.